Graphene-based photonic crystal

Oleg L. Berman[1], Vladimir S. Boyko[1], Roman Ya. Kezerashvili[1,2], Anton A. Kolesnikov[3], and Yurii E. Lozovik[3,4]
[1] *Physics Department, New York City College of Technology, The City University of New York, Brooklyn, NY 11201, USA*
[2] *The Graduate School and University Center, The City University of New York, New York, NY 10016, USA*
[3] *Institute of Spectroscopy, Russian Academy of Sciences, 142190 Troitsk, Moscow Region, Russia*
[4] *Moscow Institute of Physics and Technology (State University), 141700, Dolgoprudny, Russia*

A novel type of photonic crystal formed by embedding a periodic array of constituent stacks of alternating graphene and dielectric discs into a background dielectric medium is proposed. The photonic band structure and transmittance of such photonic crystal are calculated. The graphene-based photonic crystals can be used effectively as the frequency filters and waveguides for the far infrared region of electromagnetic spectrum. Due to substantial suppression of absorption of low-frequency radiation in doped graphene the damping and skin effect in the photonic crystal are also suppressed. The advantages of the graphene-based photonic crystal are discussed.

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Photonic crystals attract the growing interest due to various modern applications [1, 2]. For example, they can be used as the frquency filters and waveguides. Photonic crystals are media with a spatially periodical dielectric function [3–6]. This periodicity can be achieved by embedding a periodic array of constituent elements with dielectric constant ε_1 in a background medium characterized by dielectric constant ε_2. Different materials have been used for the corresponding constituent elements including dielectrics [5, 6], semiconductors, metals [7–9], and superconductors [10–13].

A novel type of 2D electron system was experimentally observed in graphene, which is a 2D honeycomb lattice of the carbon atoms that form the basic planar structure in graphite [14–16]. Due to unusual properties of the band structure, electronic properties of graphene became the object of many recent experimental and theoretical studies [14–21]. Graphene is a gapless semiconductor with massless electrons and holes which have been described as Dirac-fermions [14, 15, 22]. The unique electronic properties of graphene in a magnetic field have been studied recently [23–26]. It was shown that in infrared and at larger wavelengths transparency of graphene is defined by the fine structure constant [27]. Thus, graphene has unique optical properties. The space-time dispersion of graphene conductivity was analyzed in Ref. [28] and the optical properties of graphene were studied in Refs. [29, 30].

As mentioned above, the photonic crystals with the dielectric, metallic, semiconductor, and superconducting constituent elements have different photonic band and transmittance spectra. The dissipation of the electromagnetic wave in all these photonic crystals is different. The photonic crystals with the metallic and superconducting constituent elements can be used as the frequency filters and waveguides for the far infrared region of spectrum, while the dielectric photonic crystals can be applied for the devices only for the optical region of spectrum.

In this Letter, we consider a 2D photonic crystal formed by stacks of periodically placed graphene discs embedded into the dielectric film. The stack is formed by graphene discs placed one on top of another separated by the dielectric placed between them as shown in Fig. 1. We calculate the photonic band structure and transmittance of this graphene-based photonic crystal. We will show that the graphene-based photonic crystals can be applied for the devices for the far infrared region of spectrum.

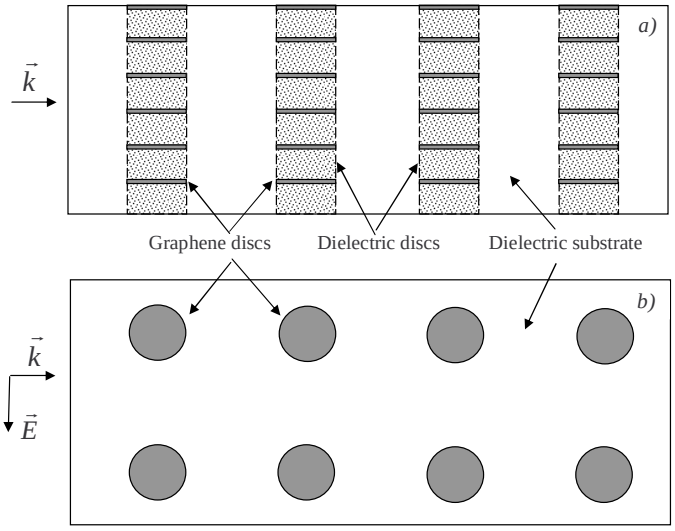

FIG. 1: Graphene-based photonic crystal: a) the side view. The material of the dielectric between graphene discs can be the same as the material of the dielectric substrate; b) the top view.

We consider polarized electromagnetic waves with the electric field **E** parallel to the plane of the graphene discs.

Expanding the electric field in terms of the Bloch waves inside a photonic crystal, one obtains from the wave equation the system of equations for Fourier components of

the electric field [6, 8]:

$$(\mathbf{k}+\mathbf{G})^2 E_{\mathbf{k}}(\mathbf{G}) = \frac{\omega^2(\mathbf{k})}{c^2}\sum_{\mathbf{G}'}\varepsilon(\mathbf{G}-\mathbf{G}')E_{\mathbf{k}}(\mathbf{G}') \ , \quad (1)$$

which presents the eigenvalue problem for finding photon dispersion curves $\omega(\mathbf{k})$. In Eq. (1) the coefficients of the Fourier expansion for the dielectric constant are given by

$$\varepsilon(\mathbf{G}-\mathbf{G}') = \varepsilon_0\delta_{\mathbf{G}\mathbf{G}'} + (\varepsilon_1-\varepsilon_0)M_{\mathbf{G}\mathbf{G}'} \ . \quad (2)$$

In Eq. (2) ε_0 is the dielectric constant of the dielectric, ε_1 is the dielectric constant of graphene multilayers separated by the dielectric material, and $M_{\mathbf{G}\mathbf{G}'}$ for the geometry considered above is

$$\begin{aligned} M_{\mathbf{G}\mathbf{G}'} &= 2f\frac{J_1(|\mathbf{G}-\mathbf{G}'|r)}{(|\mathbf{G}-\mathbf{G}'|r)} \ , \quad \mathbf{G}\neq\mathbf{G}' \ , \\ M_{\mathbf{G}\mathbf{G}'} &= f \ , \quad \mathbf{G}=\mathbf{G}' \ , \end{aligned} \quad (3)$$

where J_1 is the Bessel function of the first order, and $f = S_g/S$ is the filling factor of 2D photonic crystal.

In our consideration the size of the graphene discs was assumed to be much larger than the period of the graphene lattice, and we applied the expressions for the dielectric constant of the infinite graphene layer for the graphene discs, neglecting the effects related to their finite size.

The dielectric constant $\varepsilon_1(\omega)$ of graphene multilayers system separated by the dielectric layers with the dielectric constant ε_0 and the thickness d is given by [29, 30]

$$\varepsilon_1(\omega) = \varepsilon_0 + \frac{4\pi i\sigma_g(\omega)}{\omega d} \ , \quad (4)$$

where $\sigma_g(\omega)$ is the dynamical conductivity of doped graphene for the high frequencies ($\omega \gg kv_F$, $\omega \gg \tau^{-1}$) at temperature T given by [29, 30]

$$\begin{aligned} \sigma_g(\omega) = \frac{e^2}{4\hbar}\Big[\eta(\hbar\omega-2\mu) \\ +\frac{i}{2\pi}\left(\frac{16k_BT}{\hbar\omega}\log\left[2\cosh\left(\frac{\mu}{2k_BT}\right)\right]\right. \\ \left.-\log\frac{(\hbar\omega+2\mu)^2}{(\hbar\omega-2\mu)^2+(2k_BT)^2}\right)\Big] \ . \end{aligned} \quad (5)$$

Here e is the charge of an electron, τ^{-1} is the electron collision rate, k is the wavevector, $v_F = 10^8$ cm/s is the Fermi velocity of electrons in graphene [30], k_B is the Boltzmann constant, and μ is the the chemical potential determined by the electron concentration $n_0 = (\mu/(\hbar v_F))^2/\pi$, which is controlled by the doping. The chemical potential can be calculated as $\mu = (\pi n_0)^{1/2}\hbar v_F$. In the calculations below we assume $n_0 = 10^{11}$ cm^{-2}. For simplicity, we assume that the dielectric material is the same for the dielectric discs between the graphene disks and between the stacks. As the dielectric material we consider SiO_2 with the dielectric constant $\varepsilon_0 = 4.5$.

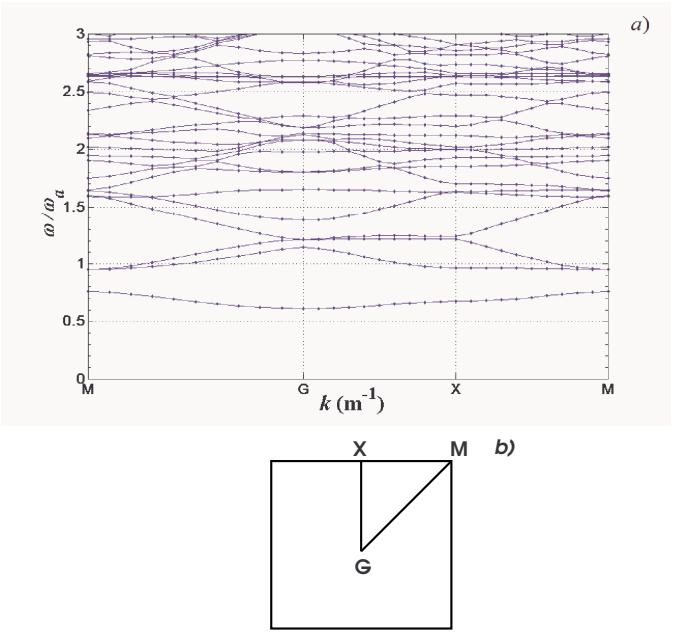

FIG. 2: a) Band structure of graphene based 2D square photonic crystal of cylinder array arranged in a square lattice. The cylinders consist of metamaterial stack of graphene monolayer discs separated by the dielectric discs. The filling factor $f = 0.3927$. M, G, X, M are points of symmetry in the first (square) Brillouin zone. b) The first Brillouin zone of the 2D photonic crystal.

To illustrate the effect let us, for example, consider the 2D square lattice formed by the graphene based metamaterial embedded in the dielectric. The photonic band structure for the graphene based 2D photonic crystal with the array of cylinders arranged in a square lattice with the filling factor $f = 0.3927$ is presented in Fig. 2. The cylinders consist of the metamaterial stacks of alternating graphene and dielectric discs. The period of photonic crystal is $a = 25$ μm, the diameter of discs is $D = 12.5$ μm, the width of the dielectric layers $d = 10^{-3}$ μm . Thus the lattice frequency is $\omega_a = 2\pi c/a = 7.54\times10^{13}$ rad/s. The results of the plane wave calculation for the graphene based photonic crystal are shown in Fig. 2, and the transmittance spectrum obtained using the Finite-Difference Time-Domain (FDTD) method [32] is presented in Fig. 3. Let us mention that plane wave computation has been made for extended photonic crystal, and FDTD calculation of the transmittance have been performed for five graphene layers. A band gap is clearly apparent in the frequency range $0 < \omega < 0.6$ and $0.75 < \omega < 0.95$ in units of $2\pi c/a$. The first gap is originated from the electronic structure of the doped graphene, which prevents absorbtion at $\hbar\omega < 2\mu$ (see also Eq. (5)). The photonic crystal structure manifests itself in the dependence of the lower photonic band on the wave vector k. In contrast, the second gap $0.75 < \omega < 0.95$ is caused by the photonic crystal structure and dielectric contrast.

According to Fig. 3, the transmittance T is almost zero

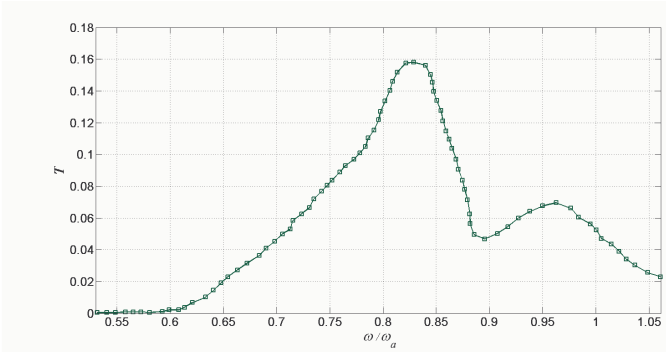

FIG. 3: The transmittance T spectrum of graphene based 2D photonic crystal.

for the frequency lower than $0.6\omega_a$, which corresponds to the first band gap shown in Fig. 2. The second gap in Fig. 2 (at the point G) corresponds to $\omega = 0.89\omega_a$, and it also corresponds to the transmittance spectrum minimum on Fig. 3.

Let us mention that at $\hbar\omega < 2\mu$ the dissipation of the electromagnetic wave in graphene is suppressed. In the long wavelength (low frequency) limit, the skin penetration depth is given by $\delta_0(\omega) = c/Re\left[2\pi\omega\sigma_g(\omega)\right]^{1/2}$ [31]. According to Eq. (5), $Re[\sigma_g(\omega < 2\mu)] = 0$, therefore, $\delta_0(\omega) \to +\infty$, and the electromagnetic wave penetrates along the graphene layer without damping. For the carrier densities $n_0 = 10^{11}$ cm^{-2} the chemical potential is $\mu = 0.022$ eV [29], and for the frequencies $\nu < \nu_0 = 10.42$ THz we have $Re[\sigma_g(\omega)] = 0$ at $\omega \gg 1/\tau$ the electromagnetic wave penetrates along the graphene layer almost without damping, which makes the graphene multilayer based photonic crystal to be distinguished from the metallic photonic crystal, where the electromagnetic wave is essentially damped. As a result, the graphene-based photonic crystals can have the sizes much larger than the metallic photonic crystals. The scattering of the electrons on the impurities can result in non-zero $Re[\sigma_g(\omega)]$, which can cause the dissipation of the electromagnetic wave. Since the electron mobility in graphene can be much higher than in typical semiconductors, one can expect that the scattering of the electrons on the impurities does not change the results significantly.

The physical properties of graphene-based photonic crystals are different from the physical properties of other types of photonic crystals, since the dielectric constant of graphene has the unique frequency dependence [29, 30]. According to the results presented above, the graphene-based photonic crystal has completely different photonic band structure in comparison to the photonic crystals based on the other materials. The photonic band structure of the photonic crystal with graphene multilayer can be tuned by changing the distance d between graphene discs in the r.h.s. of Eq. (4). The photonic band structure of the graphene-based photonic crystals can also be controlled by the doping, which determines the chemical potential μ entering the expressions for the conductivity and dielectric constant of graphene multilayer (5).

Comparing the photonic band structure for graphene-based photonic crystal presented in Fig. 2 with the dielectric [6], metallic [8, 9], semiconductor [8] and superconductor-based [11, 12] photonic crystals, we conclude that only graphene- and superconductor-based photonic crystals have essential photonic band gap at low frequencies starting $\omega = 0$, and the manifestation of the gap in the transmittance spectra is almost not suppressed by the damping effects. Therefore, only graphene-based and superconducting photonic crystals can be used effectively as the frequency filters and waveguides in low-frequency for the far infrared region of spectrum, while the devices based on the dielectric photonic crystals can be used only in the optical region of electromagnetic waves spectrum. The graphene based-photonic crystal can be used at room temperatures, while the superconductor-based photonic crystal can be used only at low temperatures below the critical temperature T_c, which is about 90 K for the YBCO superconductors.

In summary, the graphene-based photonic crystal presented in this Letter is the novel type of photonic crystal. The frequency band structure of a 2D photonic crystal with the square lattice of the metamaterial stacks of the alternating graphene and dielectric discs is obtained. The electromagnetic wave transmittance of such photonic crystal is calculated. The graphene-based photonic crystals have the following advantages that distinguish them from the other types of photonic crystals. They can be used as the frequency filters and waveguides for the far-infrared region of spectrum at the wide range of the temperatures including the room temperatures. The photonic band structure of the graphene-based photonic crystals can be controlled by changing the thickness of the dielectric layers between the graphene discs and by the doping. The sizes of the graphene-based photonic crystals can be much larger than the sizes of metallic photonic crystals due to the small dissipation of the electromagnetic wave. Let us also mention that above for simplicity we assume that the dielectric material is the same between the graphene disks and between the stacks. This assumption has some technological advantage for the most easier possible experimental realization of the graphene-based photonic crystal.

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